Maths

(Pages : 4) **V – 1683**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2025

First Degree Programme under CBCSS

Mathematics

Core Course IX

MM 1641 : REAL ANALYSIS – II

(2018 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

Answer the first **ten** questions are compulsory. Each question carries **1** mark.

1. State Divergence criterion for functional limits.

2. Determine the points of discontinuity of the greatest integer function.

3. Find $\lim_{x \to 0} \frac{\sqrt{1+2x} - \sqrt{1+3x}}{x+2x^2}$ where $x > 0$.

4. State Rolle's theorem.

5. Find the interval on which the function $f(x) = 3x - 4x^2$ is increasing.

6. Evaluate $\lim_{x \to 0} \frac{x^2 + x}{\sin 2x}$.

P.T.O.

7. When a function is Riemann integral.

8. State true or false: If $|f|$ is integrable on $[a,b]$ then f is integrable on $[a,b]$. Justify.

9. Find the measure of the set $A=\left\{1,\frac{1}{2},\frac{1}{2^2},\cdots\frac{1}{2^{100}}\right\}$.

10. Give an example of a continuous function which is not differentiable.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions from this Section. Each question carries **2** marks.

11. Using $\in\rightarrow\delta$ version of the limit of function, show that $\lim\limits_{x\to 2} x^2=4$.

12. Discuss the continuity of the function $g(x)=\begin{cases} x\sin(1/x), & if\ x\neq 0 \\ 0 & if\ x=0 \end{cases}$.

13. State Non-Uniform Continuity criterion.

14. Let $f:A\rightarrow\mathbb{R}$ and $g:B\rightarrow\mathbb{R}$ where $f(A)\subseteq B$. If f and g are differentiable then show that its composition $g\circ f$ is also differentiable.

15. If $g:A\rightarrow\mathbb{R}$ is differentiable on an interval A and satisfies $g'(x)=0$ for all $x\in A$, then prove that $g(x)=k$ for some constant $k\in\mathbb{R}$.

16. Show that if f is differentiable on an interval with $f'(x)\neq 1$, then there exist at most one point x such that $f(x)=x$.

17. If $\lim\limits_{x\to a} f(x)$ and $\lim\limits_{x\to a} f(x)g(x)$ both exist, then does it follow that $\lim\limits_{x\to a} g(x)$ exists? Justify your answer.

18. Show that the greatest integer function $[[x]]$ is not differentiable at $x=1$.

19. Compare $U(f,P)$ for the function f defined by $f(x)=x^2$ on $[0,1]$ and $P=\left\{0,\frac{1}{4},\frac{2}{4},\frac{3}{4},1\right\}$.

20. Show that the Dirichlet's function is not integrable on $[0,1]$.

21. Prove that if f and g are integrable on $[a,b]$ then $f+g$ is integrable on $[a,b]$.

22. If f is continuous on $[a,b]$, show that there exists a point $c\in(a,b)$ such that $f(c)=\frac{1}{b-a}\int_a^b g(t)\,dt$.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions from this section. Each question carries **4** marks.

23. Show that $\lim_{x\to a} g(x)$ does not exist where $g(x)=\begin{cases}1 & \text{if } x\in Q\\ 0 & \text{if } x\in Q\end{cases}$.

24. Prove that the composition of continuous functions is again continuous.

25. Show that a real valued continuous function on a compact set $K\subseteq\mathbb{R}$ is uniformly continuous on K.

26. State and prove Darboux's theorem.

27. Let $f:I\to\mathbb{R}$ be bounded on I. If Q is a refinement of a partition of P of I then show that $U(f,P)\geq U(f,Q)$.

28. Prove that if f is continuous on $[a,b]$, then f is integrable on $[a,b]$.

29. Show that the function $f(x)=\begin{cases}1 & if\ x \neq 1 \\ 0 & if\ x=1\end{cases}$ is integrable on $[0,2]$.

30. Let $f:[a,b]\to\mathbb{R}$ be bounded on $[a,b]$ and let $c\in(a,b)$. Prove that f is integrable on $[a,b]$ if and only if f is integrable on $[a,c]$ and $[c,b]$.

31. Show that if $f:A\to\mathbb{R}$ is uniformly continuous on A and (x_n) is a Cauchy sequence in A then $(f(x_n))$ is a Cauchy sequence.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions from this section. Each question carries **15** marks.

32. (a) Let $f:A\to\mathbb{R}$ be continuous. If $K\subseteq A$ is compact, prove that $f(K)$ is compact. **7**

(b) Show that $f(x)=1/x^2$ is uniformly continuous on the set $[1,\infty]$ but not on the set $(0,1)$. **8**

33. State and prove Intermediate Value theorem. Is the converse true. Justify.

34. (a) Prove that $\lim_{x\to 0}\cos(1/x)$ does not exist but that $\lim_{x\to 0}x\cos(1/x)=0$. **7**

(b) State and prove Riemann's criterion for integrability. **8**

35. (a) Let $f:[a,b]\to\mathbb{R}$ be increasing on $[a,b]$. Show that f is integrable on $[a,b]$. **7**

(b) Let $g:[a,b]\to\mathbb{R}$ be continuous on $[a,b]$. Prove that $G(x)=\int_a^x g(t)\,dt$ is differentiable on $[a,b]$ and $G'(x)=g(x)$ for all $x\in[a,b]$. **8**

(2 × 15 = 30 Marks)

(Pages : 4)

V – 1685

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2025

First Degree Programme under CBCSS

Mathematics

Core Course X

MM 1642 : COMPLEX ANALYSIS II

(2021 Admission Onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – I

Answer **all** questions.

1. State Cauchy's integral formula.

2. State generalized Cauchy's integral formula.

3. Evaluate $\int_{|z|=3} \frac{z^2}{z-2} dz$.

4. When we say that a series is convergent? Give an example of a convergent series.

5. State True or False. "If a sequence is pointwise convergent then it is uniformly convergent".

P.T.O.

6. Using the ratio test, check the convergence of $\sum_{j=1}^{\infty} \frac{1}{j!}$.

7. What is Maclaurin's series?

8. State Cauchy Residue theorem.

9. Give an example of a rational function.

10. Find the zeros of $f(z) = (z-2)/z^2$.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions.

11. State and prove Morera's theorem.

12. Find $\int_{|z|=3} \frac{e^z}{z-2} dz$.

13. Show that $\int_C \frac{dz}{z-1} = 2\pi i$ where C is the circle $|z| = 3$.

14. Show that $\sum_{j=0}^{\infty} c^j$ converges to $\frac{1}{1-c}$ if $|c| < 1$.

15. Show that the series $\sum_{j=0}^{\infty} \frac{4^j}{j!}$ converges.

16. If f_n is a sequence of functions analytic in a simply connected domain D and converges uniformly to f in D then prove that f is analytic in D.

17. Find the Laurent series for the function $\frac{z^2 - 2z + 3}{z-2}$ in the region $|z-1| > 1$.

18. Compute the residue of $f(z) = \cot z$ at each singularity.

19. Find the residue of $f(z) = 1/(z+1)^3$ at its pole.

20. Evaluate $\int_{|z|=2} \frac{dz}{z^3(z-1)}$.

21. If z_0 is a pole of $f(z)$ then show that $\lim_{z \to z_0} f(z) = \infty$.

22. Find the Taylor series expansion of e^z at $z = 1$.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions.

23. Find $\int_C \frac{z^2 e^z}{2z+i} dz$ where C is the circle $|z| = 1$.

24. If F is analytic in a domain D, show that all its derivatives exist and are analytic in D.

25. If $f = u + iv$ is analytic in a domain D then show that all partial derivatives of u and v are continuous in D.

26. Show that the series $\sum_{j=0}^{\infty} \frac{3+2i}{(j+1)^j}$ converges.

27. Find a function f that is analytic and satisfies the differential equations $\frac{df(z)}{dz} = 3if(z)$.

28. Find the radii of convergence of the following power series

(a) $\sum_{n=0}^{\infty} \frac{z^n}{n!}$

(b) $\sum_{n=1}^{\infty} \frac{n!}{n^n} z^n$

29. If f is a pole of order m then show that

$$\text{Res}(f, z_0) = \lim_{z \to z_0} \frac{1}{(m-1)!} \frac{d^{m-1}}{dz^{m-1}}[(z-z_0)^m f(z)].$$

30. Find $PV \int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)^2} dx$.

31. Evaluate $\int_{|z|=2} \frac{2z^2+z}{z^2-1} dz$ using Cauchy Residue theorem.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions.

32. (a) If $f(z)$ is analytic inside and on a closed contour C and z_0 is a point inside C then show that $f'(z_0) = \frac{1}{2\pi i} \int \frac{f(z)}{(z-z_0)^2} dz$.

(b) Find $\int_{|z|=3} \frac{z^2+5}{(z-2)^2} dz$.

33. Find the Laurent series expansion of $\frac{1}{(z-1)(z-2)}$ in the following regions.

(a) $|z| < 1$

(b) $1 < |z| < 1$

(c) $|z| > 2$

34. (a) Explain different types of singularities with examples.

(b) Find the residue at $z=0$ of $f(z) = ze^{3/z}$.

35. Evaluate $\int_0^{2\pi} \frac{\sin^2\theta}{5+4\cos\theta} d\theta$.

(2 × 15 = 30 Marks)

(Pages : 4) **V – 1686**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2025

First Degree Programme under CBCSS

Mathematics

Core Course XI

MM 1643 : ABSTRACT ALGEBRA-RING THEORY

(2018 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

Answer **all** questions. Each carries **1** mark.

1. Is $\left\{\begin{pmatrix} 0 & a \\ 1 & 0 \end{pmatrix} : a \in Z\right\}$ is a subring of $M_2(Z)$? Why?

2. Give example for a zero divisor in Z_9 and show that the given element is indeed a zero divisor.

3. For a ring R, ideal A in it, R/A is an integral domain implies that A is a ———— ideal.

4. Give an example for a maximal ideal in Z.

5. What is the natural homomorphism from Z to Z_{15}?

6. Give an isomorphism from Z to itself.

7. State Eisenstein's criterion.

8. Define prime in an integral domain.

9. If F is a field, then $F[x]$ is a Euclidean Domain with d defined by ————.

10. In an integral domain, if $\langle a \rangle = \langle b \rangle$, then is it necessary that $a = b$? Justify.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions. Each carries **2** marks.

11. Is $S = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : ab - bc \neq 0 \right\}$ a subring of $M_2(Z)$, the set of all 2×2 integer matrices? Justify.

12. Define characteristic of a ring. What is the characteristic of Z_8?

13. In $Z / 4Z$, using standard operations that makes it a ring, explain the addition and multiplication of elements in it using two nonzero elements.

14. Is the ideal generated by $x^2 + 1$ in $Z_2[x]$ prime? Why?

15. Is $2Z$ isomorphic to $3Z$? Justify.

16. If R is a commutative ring, show that characteristic of R and $R[x]$ are the same.

17. Find the kernel of the ring homomorphism $k \to k \bmod 10$ from Z to Z_{10}.

18. Find all monic irreducible polynomials of degree 2 over Z_3.

19. If d is defined on $Z[i]$ by $d(a + ib) = a^2 + b^2$, show that $d(x) \leq d(xy)$ for any $x, y \in Z[i]$.

20. Define Euclidean domain. Show that Z is a Euclidean domain.

21. In an integral domain, show that the product of an irreducible element and unit is irreducible.

22. Give two factorizations of 10 in $Z\left[\sqrt{-6}\right]$.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. Each carries **4** marks.

23. Show that $\{a + ib : a, b \in Z\}$ is a subring of C, the set of all complex numbers. Is this an ideal? Justify.

24. Show that a finite integral domain is a field.

25. Show that the ideal generated by $x^2 + 1$ is maximal in $R[x]$ where R is the set of all real numbers.

26. Prove that if R is a ring with unity, then $f(n) = n.1$ is a ring homomorphism from Z, the ring of integers to R.

27. If F is a field, show that $F[x]$ is a PID.

28. Show that the pth cyclotomic polynomial is irreducible over the rationals Q.

29. Prove that $Z[i]$ is a Euclidean Domain with $d(a + ib) = a^2 + b^2$.

30. Let $f(x) = a_n x^n + \ldots + a_0 \in Z[x]$ and suppose that p is prime such that $p \nmid a_n, p \mid a_{n-1}, \cdots$ and $p^2 \nmid a_0$. Prove that f is irreducible over rationals Q.

31. In a principal ideal domain, show that if an element is irreducible then it is a prime.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. Each question carries **15** marks.

32. (a) Let R be a ring with unity. Show that if 1 has infinite additive order, then characteristic of R is 0, and otherwise it is n where n is the additive order of 1.

    (b) Show that the characteristic of an integral domain is 0 or a prime.

    (c) Find the characteristic of $\{0, 2, 4, \ldots, 10\}$ as a subring of Z_{10}.

33. (a) Let F be a field, $p(x) \in F[x]$. Prove that $\langle p(x) \rangle$ is a maximal ideal if and only if $p(x)$ is irreducible over $F[x]$.

    (b) Construct a field with 8 elements.

34. (a) State and prove the division algorithm for $F[x]$.

    (b) Use this to find the quotient and remainder upon dividing $3x^4 + x^3 + 2x^2 + 1$ by $x^2 + 4x + 2$.

35. (a) Prove that every Euclidean domain is a PID.

    (b) Show that the integral domain $Z[\sqrt{-5}]$ is not a UFD.

(2 × 15 = 30 Marks)

(Pages : 6) **V – 1688**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2025

First Degree Programme under CBCSS

Mathematics

Core Course XII

MM 1644 : LINEAR ALGEBRA

(2021 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. Each question carries **1** mark.

1. Write the following system of equations as a single vector equation $x-2y=1$, $x+3y=2$.

2. For which values of β, is there a whole line of solution for the following equation

$$\beta x+2y=1$$
$$2x+4\beta y=2$$

3. If $A=\begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \end{bmatrix}$ and $B=\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ then express AB as the linear combination of columns of A.

P.T.O.

4. Give an example of a subspace of $\mathbb{R}^3$ over $\mathbb{R}$.

5. Define basis of a vector space.

6. Find the null space of the matrix $A=\begin{bmatrix} 2 & -1 \\ -4 & 2 \end{bmatrix}$.

7. Find the rank of the matrix $\begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 0.5 & -0.5 & 0.5 \end{bmatrix}$.

8. Find the determinant of the matrix $\begin{bmatrix} 1 & -2 & 1 \\ 2 & -4 & 1 \\ -1 & 2 & 7 \end{bmatrix}$.

9. Find the Eigen values of $\begin{bmatrix} 4 & 7 \\ 0 & -2 \end{bmatrix}$.

10. Write the sum of all the three Eigen values of the matrix $A=\begin{bmatrix} 1 & -1 & 1 \\ 1 & 2 & 0 \\ 3 & 1 & 3 \end{bmatrix}$.

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. Each question carries **2** marks.

11. What multiple ℓ_{21} of equation 1 should be subtracted from equation 2.

$$2x+3y=1$$
$$10x+9y=11$$

After this elimination step, write down the upper triangular system and circle the two pivots.

12. Let $I=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$, then find E_{31} with $\ell=4$.

13. If A is a 7×9 matrix with a two-dimensional null space, what is the rank of A?

14. Define Non Singular matrix and verify whether $A=\begin{bmatrix}2 & -4 & 4\\1 & 3 & -7\\-1 & 2 & -2\end{bmatrix}$ is non singular or not?

15. Let $v_1, v_2,....,v_n$ are linearly independent vectors in a vector space V, What is the dimension of the span of these vectors? If we add a vector v_{n+1} where v_{n+1} is a linear combination of $v_1, v_2,....,v_n$, then what the change in the dimension of the span of these $(n+1)$ vectors?

16. Write down the 2 by 2 matrices A and B that have entries $a_{ij}=i-j$ and $b_{ij}=i+j$ multiply them to find, AB and $|AB|$.

17. Check whether the set $W=\{(x,y)\in\mathbb{R}^2;\ 2x+y=0\}$ is a subspace of $\mathbb{R}^2$ or not?

18. Check whether the columns of the matrix $A=\begin{bmatrix}2 & 4 & 3\\1 & 2 & 1\\-1 & -2 & -1\end{bmatrix}$ are linearly independent or not?

19. Using Crammers Rule solve $2x_1+x_2=3, x_1-4x_2=-3$.

20. Draw the triangle with vertices $A=(1,1), B=(2,2)$, and $C=(0,2)$. Find its area.

21. Find the Eigen values of the matrix A and A^2 if $A=\begin{bmatrix} 2 & -4 \\ -1 & 2 \end{bmatrix}$.

22. Give examples of A and B such that $A+B$ is invertible although A and B are not invertible.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. Each question carries **4** marks.

23. Test the consistency and solve :

$x-2y+z=0; 2y-8z=8; -4x+5y+9z=-9$

24. Row reduce the matrix A below to echelon form, and locate the pivot columns of

$$A=\begin{bmatrix} 0 & -3 & -6 & 4 & 9 \\ -1 & -2 & -1 & 3 & 1 \\ -2 & -3 & 0 & 3 & -1 \\ 1 & 4 & 5 & -9 & -7 \end{bmatrix}.$$

25. Find L and U such that $LU=A$; where $A=\begin{bmatrix} 2 & 5 \\ -1 & 1 \end{bmatrix}$.

26. Find the Eigen values of the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$.

27. Consider the Linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x,y)=(3y,2x)$. Let S be the unit circle in $\mathbb{R}^2$ given by $S=\{(x,y):+x^2+y^2=1\}$. Describe the set $T(S)$.

28. Show that $S=\{(1,2,1),(1,1,0),(1,0,0)$ form a basis of $\mathbb{R}^3$.

29. Use the Gauss-Jordan Method to find A^{-1} if

$$A=\begin{bmatrix} 2 & 1 & 1 \\ 4 & -6 & 0 \\ -2 & 7 & 2 \end{bmatrix}.$$

30. If λ is a Eigen value of a invertible matrix A, then prove that $\frac{1}{\lambda}$ is a Eigen value of A^{-1}.

31. Prove that the Eigen values of a real symmetric matrix are real numbers

($6 \times 4 = 24$ Marks)

SECTION – D

Answer any **two** questions. Each question carries **15** marks.

32. (a) Prove that if A and B are invertible matrices, then $(AB)^{-1}=B^{-1}A^{-1}$

(b) If $A=\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and $B=\begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}$ then verify $(AB)^{-1}=B^{-1}A^{-1}$

33. (a) Given u and v in a vector space V. Show that $Span(u,v)$ is a subspace of V.

(b) Let H be the set of all vectors the form $(a-3b, b-a, a, b)$ where a and b are arbitrary scalars. Show that H is a subspace of $\mathbb{R}^4$.

34. (a) Solve the system $x_2 + 5x_3 = -4, x_1 + 4x_2 + 3x_3 = -2, 2x_1 + 7x_2 + x_3 = -2$.

(b) Do the three planes $2x_1 + 4x_2 + 4x_3 = 4$, $x_2 - 2x_3 = -2$, $2x_1 + 3x_2 = 0$ have atleast one common point of intersection? Explain.

35. Diagonalize the matrix $\begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

(2 × 15 = 30 Marks)

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(Pages : 4) **V – 1689**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2025

First Degree Programme under CBCSS

Mathematics

Core Course XIII

MM 1645 : INTEGRAL TRANSFORMS

(2018 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

Answer **all** questions. They carry **1** mark.

1. Find $\mathcal{L}\{\sin 2t\}$.

2. Write the Linearity property of Laplace Transform.

3. Using $\mathcal{L}\{f(t)\}$ and $\mathcal{L}\{f'(t)\}$, write the formula for $f(0)$.

4. Define Unit step function $u(t-a)$.

5. Find $\mathcal{L}^{-1}\left\{\frac{1}{(s-2)^2}\right\}$.

6. Find the period of $f(x) = \sin 4x$.

7. Define Odd Function with an example.

P.T.O.

8. Write the Fourier Integral representation of a continuous function $f(x)$.

9. Write the Fourier cosine transform of $e^{-ax}, a > 0$.

10. Define Fourier transform of a function $f(x)$.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions. These question carries **2** marks each.

11. Using the definition, find Laplace transform of e^{at}.

12. Find $\mathcal{L}\{t^2 + 3\cos 2t + e^{4t}\}$.

13. State first shifting formula on Laplace transform, hence find $\mathcal{L}\{e^t t\}$.

14. Find $\mathcal{L}^{-1}\left\{\frac{1}{s(s-1)^2}\right\}$.

15. If $\mathcal{L}\{f(t)\} = F(s)$ then find $\mathcal{L}\{f'(t)\}$, where $f'(t) = \frac{df}{dt}$ and verify the same with $f(t) = \sin t$.

16. Using the concept of Laplace transform find $\int_0^{\infty} e^{-3t}\, t \sin t \, dt$.

17. Find the Laplace transform of Unit step function $u(t-a)$.

18. Find the Fourier coefficient b_n of the function $f(x) = \begin{cases} 0 & if -\pi \le x \le 0 \\ 1 & 0 \le x \le \pi \end{cases}$.

19. Find the Fourier transform of $f(x) = \begin{cases} e^{-ax} & , x > 0 \\ 0, & x < 0 \end{cases}; a > 0$.

20. State and prove Linearity property of Fourier transform.

21. Find Fourier Cosine transform of $f(x)=\begin{cases} k, & 0<x<a \\ 0, & x>0 \end{cases}$.

22. Show that $\mathcal{F}_s\{f'(x)\}=-\omega\,\mathcal{F}_c\{f(x)\}$

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. These question carries **4** marks each.

23. Find $\mathcal{L}\{\sin^3 2t\}$.

24. Find the Laplace transform of $\dfrac{e^{-at}-e^{-bt}}{t}$.

25. Using Convolution property find $\mathcal{L}^{-1}\left\{\dfrac{s}{(s^2+1)^2}\right\}$.

26. Define Dirac's Delta function and find its Laplace transform.

27. Find $\mathcal{L}^{-1}\left\{\ln\dfrac{s^2+\omega^2}{s^2}\right\}$.

28. Represent function $f(x)=|x|; -\pi\le x\le\pi$ as a Fourier series.

29. Find the Fourier Transform of $f(x)=1$ *if* $|x|<1$ and $f(x)=0$ *otherwise.*

30. Find half range Fourier sine of $f(x)=\begin{cases} \dfrac{2k}{L}x & \text{if } 0<x<\dfrac{L}{2} \\ \dfrac{2k}{L}(L-x) & \text{if } \dfrac{L}{2}<x<L \end{cases}$.

31. Find Fourier integral representation of function of $f(x)=\begin{cases} 1 & \text{if } |x|<1 \\ 0 & \text{if } |x|>1 \end{cases}$.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. These question carries **15** marks each.

32. (a) Using Laplace transform, solve the differential equation $y'' + 4y = 4x$. Give the initial conditions $y(0) = 1$ and $y'(0) = 5$.

    (b) Using convolution property of Laplace transform solve the integral equation $y(x) = x^3 + \int_0^x \sin(x-t)\,y(t)\,dt$.

33. (a) Deduce a formula to calculate the Laplace transform of the n^{th} derivative $f^n(t)$ of a function $f(t)$.

    (b) Using Laplace Transform Solve the system of differential equations $y_1' + y_1 = 4y_2$ and $3y_1 - y_2' = 2y_2$ given $y_1(0) = 3, y_2(0) = 4$.

34. Find the Fourier series of 2-periodic function $f(x) = \begin{cases} 1 & \text{for } 0 \le x \le 1 \\ x & \text{for } 1 \le x \le 2 \end{cases}$ Hence deduce that

    (a) $\sum_1^\infty \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$.

    (b) $\sum_1^\infty \frac{(-1)^{n+1}}{2n-1} = \frac{\pi}{4}$.

35. Using Fourier integral representation, show that $\int_0^\infty \frac{\cos\frac{\pi\omega}{2}}{1-\omega^2} \cos\omega x\, d\omega = \begin{cases} \frac{\pi}{2}\cos x & ; |x| \le \frac{\pi}{2} \\ 0 & ; |x| > \frac{\pi}{2} \end{cases}$.

(2 × 15 = 30 Marks)

(Pages : 4) **V – 1690**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2025

First Degree Programme under CBCSS

Mathematics

Elective Course

MM 1661.1 : GRAPH THEORY

(2018 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

Answer **all** questions. They carry **1** mark each.

1. Define simple graph.
2. Draw the complete bipartite graph $K_{2,2}$
3. The complete graph K_5 is ___________ regular.
4. Define a connected graph.
5. State Whitney's theorem.
6. A graph is called Eulerian if ___________.
7. Give an example of a non planar graph.
8. What is a Hamiltonian cycle?
9. Find the number of faces of K_3.
10. A connected graph G is Euler if and only if the degree of every vertex is ___________.

(10 × 1 = 10 Marks)

P.T.O.

SECTION – II

Answer **any eight** questions. These questions carry **2** marks each.

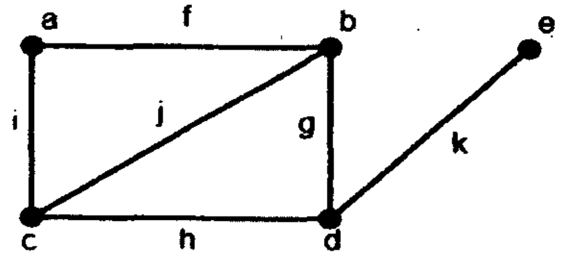
11. Prove that in any graph G, there is an even number of odd vertices.

12. Draw a graph G with adjacency matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$.

13. Prove that any tree T with at least two vertices has more than one vertex of degree one.

14. Define graph isomorphism and give an example of two isomorphic graphs.

15. Consider the following graph G.



Draw G-k and G-b.

16. Define cycle in a graph and draw C_5.

17. Check whether the complete graph K_3 is Euler or not.

18. State Chinese Postman theorem.

19. Define bridge of a graph and find the number of components of a connected graph.

20. State Kuratowski's theorem.

21. Prove that if G is a simple planar graph, then G has a vertex v with $d(v) \leq 5$.

22. Explain Platonic bodies.

(8 × 2 = 16 Marks)

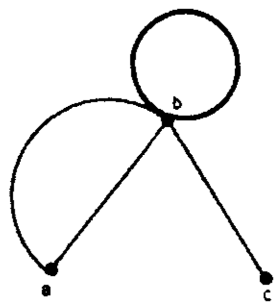
SECTION – III

Answer **any six** questions. These questions carry **4** marks each.

23. Suppose that we have three houses each of which have to be supplied with electricity, gas and water. Is it possible to connect each utility with each of the three houses without the lines or mains crossing?

24. Define complete graph and draw complete graph with six vertices.

25. Find the number of walks of length 2 from vertex a to vertex c in the following graph:



26. Prove that for an acyclic graph with n vertices and k connected components, there are n-k edges.

27. Explain Konigsberg bridge problem and represent the problem by a graph.

28. Prove that for a simple graph G with atleast three vertices, if the closure is complete, then G is Hamiltonian.

29. Show that K_5 is non-planar.

30. Draw an example of a simple plane graph in which degree of every vertex is at least 3.

31. Find the number of vertices of a plane graph with 5 edges and 2 faces and draw the graph.

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions. These questions carry **15** marks each.

32. Prove that a nonempty graph with atleast two vertices is bipartite if and only if it has no odd cycles.

33. Prove that if a simple graph with atleast three vertices is 2-connected if and only if for each pair of distinct vertices *u* and *v* of *G*, there are two internally disjoint *u*-*v* paths in *G*.

34. Prove that if *G* is a simple graph with atleast three vertices and the degree of every vertex is greater than or equal to *n*/2, then *G* is Hamiltonian.

35. State and prove Euler's formula.

(2 × 15 = 30 Marks)